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BRIEF COMMUNICATION

THERMOCAPILLARY INTERACTION BETWEEN A SOLID PARTICLE AND A GAS BUBBLE†

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1. INTRODUCTION

Thermocapillary motion of bubbles and drops driven by an externally imposed thermal gradient, described first by Young *et al.* (1959), has been attracting wide attention during last 15 years especially in application to material processing under conditions of microgravity (Subramanian 1983; Rogers & Davis 1990). The physical cause of a thermocapillary migration of liquid and gas particles is the mobility of an interface which brings about the motion of drops and bubbles under the action of surface tension gradients in a non-uniform temperature field. A solid particle or, say, a gas bubble with an interface frozen by adsorbed surfactants do not undergo thermocapillary migration in an externally imposed thermal gradient since there is nothing to generate flow in the ambient fluid. However, if a solid particle is hot and is placed near a *free* liquid–gas or liquid–liquid interface, a non-uniform temperature distribution around the particle induces surface tension gradients resulting in a thermocapillary flow. The latter leads to the motion of the solid particle itself.

In the present paper the spontaneous motion of a solid particle driven by the described thermocapillary interaction with a gas bubble is studied. The applied analysis uses some earlier results on the interaction of bubbles migrating in an external thermal gradient (Meyyappan *et al.* 1983), as well as the theory of the hydrodynamic interaction of two drops by Haber *et al.* (1973).

2. STATEMENT OF THE PROBLEM

Consider a solid particle with a radius a and a gas bubble with a radius b submerged in an unbounded fluid resting and thermally uniform at infinity. The solid particle is hotter or colder than the outer fluid, and it has a constant temperature T_s. The heat flux at the surface of the bubble is neglected due to the low heat conductivity of the gases.

All physical properties of the ambient fluid are assumed to be constant, except the surface tension of the bubble, which depends linearly on temperature: $\sigma = \sigma_0 + (\partial\sigma/\partial T)(T - T_0)$, where σ_0 is the surface tension corresponding to the temperature T_0 far away from the particles, and $\partial\sigma/\partial T = \text{const}$. The surface tension σ_0 is supposed to be sufficiently large to preserve the spherical shape of the bubble.

A non-uniform temperature field around the solid particle generates the surface tension gradient at the bubble surface. This gradient results in a thermocapillary flow of the contiguous fluid which causes the motion of both the bubble and the solid particle. The effect of gravity is neglected and only the motion driven by the surface tension gradient is studied. The viscosity of the gas inside the bubble is also ignored and only the flow in the ambient fluid is considered. The motion of the fluid is assumed to be sufficiently slow so that, at any moment, inertia effects are negligible and

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the Stokes approximation holds. The velocities of the bubble and of the solid particle, as yet unknown, should be found from the condition that the total forces acting on both the particle and the bubble are zero. Heat transfer is presumed to be controlled by thermal diffusion (the Peclet number is zero). The heat capacity of the solid particle is supposed to be large in comparison with that of the surrounding fluid so that the temperature of the particle does not change significantly during the characteristic time of the motion.

The following scaling is chosen: a for length, $\eta^{-1}(\partial\sigma/\partial T)(T_s - T_0)$ for velocity, where η is the viscosity of the contiguous fluid, and the dimensionless temperature is introduced, $\theta = (T - T_0)/(T_s - T_0)$.

The bipolar co-ordinate system (ξ, ζ) connected with the solid particle and the gas bubble is chosen, the surface of the solid particle corresponding to $\xi = \alpha > 0$ and the bubble surface relating to $\xi = -\beta < 0$. The ratio of the radii is then $r = b/a = \sinh\alpha/\sinh\beta$ and the separation distance between the bubble and the particle is $d = \cosh\alpha - 1 + r(\cosh\beta - 1)$.

According to the described statement, the flow and temperature fields are described by the following equations and boundary conditions

$$E^2(E^2\psi) = 0; \quad \Delta\theta = 0; \qquad [1]$$

$$\xi = 0, \quad \zeta = 0, \quad \psi/\rho^2 = 0, \quad \theta = 0; \qquad [2]$$

$$\xi = \alpha, \quad \psi = -V_s\frac{(1-\mu^2)}{2h^2}, \quad \frac{\partial\psi}{\partial\xi} = V_s\frac{(1-\mu^2)\sinh\xi}{ch^3}, \quad \theta = 1; \qquad [3]$$

$$\xi = -\beta, \quad \psi = -V_b\frac{(1-\mu^2)}{2h^2}, \quad \frac{\partial\theta}{\partial\xi} = 0, \quad \Pi_{\xi\zeta} = -h\frac{\partial\theta}{\partial\zeta}. \qquad [4]$$

Here ψ is the stream function, E^2 and Δ are the Stokes and the Laplace operators in the bispherical co-ordinate system, respectively, $h = (\cosh\xi - \mu)/c$, $\mu \equiv \cos\zeta$, $c = \sinh\alpha$, ρ is the radial co-ordinate of the related cylindrical co-ordinate system (ρ, z) [see Happel & Brenner (1965) for details]; V_s and V_b are as yet unknown projections of the dimensionless velocities of the solid particle and of the gas bubble on the z-axis. The last boundary condition in [4] is the balance of the tangential stresses at the surface of the bubble which links the temperature and the velocity fields, $\Pi_{\xi\zeta}$ being the tangential component of the viscous stress tensor.

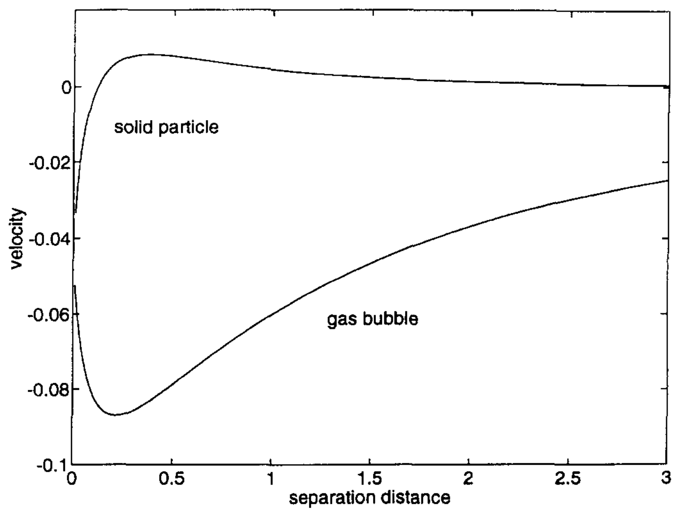

Figure 1. Dimensionless velocities of the thermocapillary motion of the solid particle and the gas bubble versus dimensionless separation distance.

3. RESULTS AND DISCUSSION

The solution of the problem [1–4] is analogous to that described in detail by Meyyappan *et al.* (1983) for two bubbles migrating in an externally imposed thermal gradient. First, the temperature field is found, the problem being reduced to an infinite linear three-diagonal system for unknown coefficients of the temperature field expansion in Legendre polynomials, the solution of which can be easily found numerically. Then, the problem for the flow field is solved and the obtained solution for the temperature field is used in the tangential stress balance. The force balance leads finally to a linear algebraic system for the velocities V_s and V_b [see Meyyappan *et al.* (1983) and Haber *et al.* (1973) for details].

Figure 1 presents the dependence of the dimensionless velocities V_s and V_b on the dimensionless separation distance d at a fixed value of the radii ratio. When the distance between the bubble and the particle is sufficiently large, their velocities are directed oppositely. For most liquids, surface tension decreases with temperature, $\partial\sigma/\partial T < 0$, and if $V_s > 0$, $V_b < 0$, this corresponds to the motion of the bubble and the *hot* solid particle *towards* each other (they move *away* from each other if the solid particle is *colder* than the outer fluid).

The flow pattern relating to this motion is shown in figure 2(a). Surface tension gradients at the liquid–gas interface induced by the hot solid particle push the fluid out of the gap between the particle and the bubble leading to their attraction. The velocities of both the solid particle and the gas bubble decrease with increase of the separation distance, however, the solid particle slows down with the increase of the separation distance much faster than the bubble. At a large distance between the bubble and the particle, the bubble velocity tends to that calculated by Young *et al.* (1959) and decreases proportionally to the temperature gradient: $V_b \propto d^{-2}$. The velocity of the solid particle V_s decreases as the velocity of the flow far from a thermocapillary migrating bubble, namely, as $V_b(b/d)^3$. In our case, however, the bubble velocity V_b itself decreases as d^{-2}, thus $V_s \propto d^{-5}$.

While the solid particle and the bubble approach each other, their velocities grow and reach maximum values at some separation distances, see figure 1 (their relative velocity also reaches a maximum value at a certain separation distance). During further approach, the velocities of the bubble and the solid particle begin to decrease because of two mechanisms hindering their motion towards each other. One is the viscous shear stresses growing with the thinning of the gap between the particle and the bubble. The other mechanism is the pressure gradient in the z-direction produced by the *translational* motion of the bubble towards the solid particle and tending to push the latter *away* from the bubble. At a sufficiently small distance, this pressure gradient becomes so strong that the solid particle stops and after that the approaching gas bubble begins to push the particle in front of itself in the same direction. This corresponds to $V_s < 0$, $V_b < 0$ in figure 1.

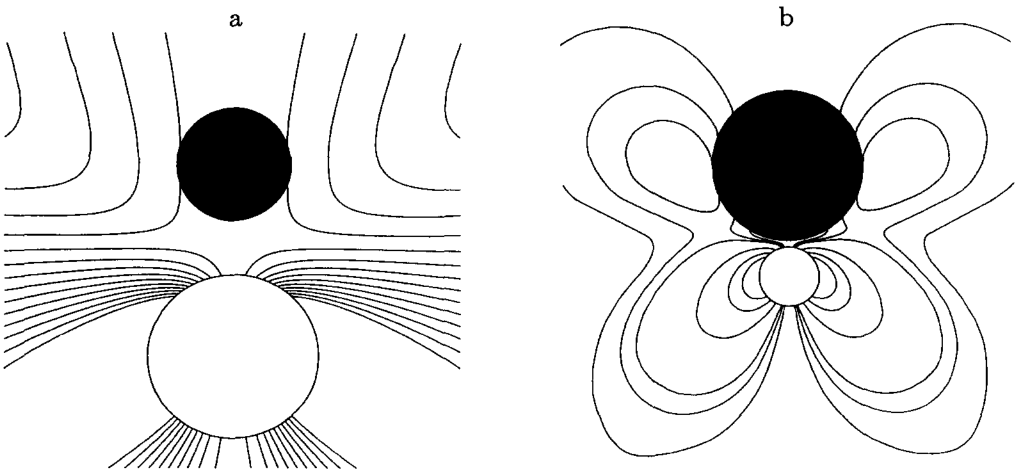

Figure 2. Thermocapillary flow (streamlines) in the surrounding fluid generated by the hot solid particle interacting with the gas bubble. (a) The bubble and the particle move towards each other, $d = 1$, $r = 1.5$ and (b) the gas bubble moves towards the hot solid particle and pushes the latter in front of itself in the same direction, $d = 0.1$, $r = 0.4$.

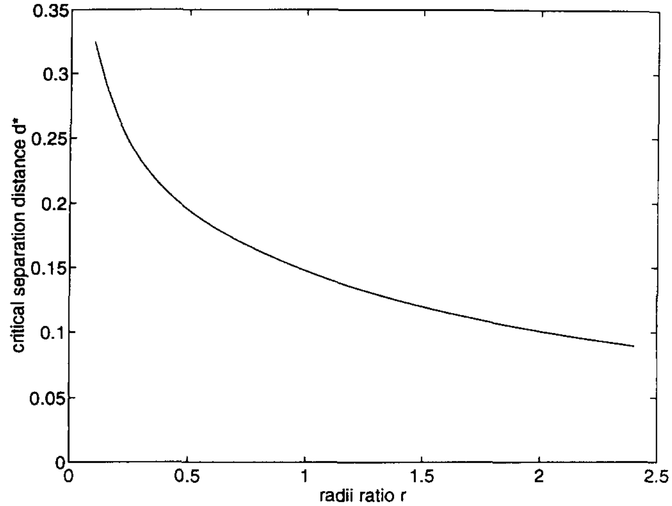

Figure 3. Critical separation distance at which the gas bubble begins to push the solid particle in front of itself, as a function of the bubble-to-particle radii ratio.

The corresponding flow pattern is presented in figure 2(b). In this case the velocity of the bubble is always larger than the velocity of the solid particle, so that while moving in the same direction, the bubble and the particle still approach each other, moving asymptotically with equal velocities in the same direction as a whole. However, at close contact, the motion of the particles cannot be described within the framework of the bispherical co-ordinate system.

The value of the critical separation distance d^* at which the solid particle stops and begins to move together with the bubble in the same direction depends on the radii ratio r. This dependence is presented in figure 3. This distance decreases with r since the surface of the large bubble is flatter in the region of the gap between the bubble and the solid particle, and the smaller the z-component of the flow and the translational velocity of the bubble. When the radius of the bubble tends to zero, the critical distance infinitely grows because in this case the thermocapillary flow is concentrated in the vicinity of the particles line of centres and is always directed from the bubble to the particle pushing the latter away from it.

The dependence of the velocities of the solid particle and the gas bubble on the ratio of their radii at a given separation distance is shown in figure 4. The velocity of a small bubble increases

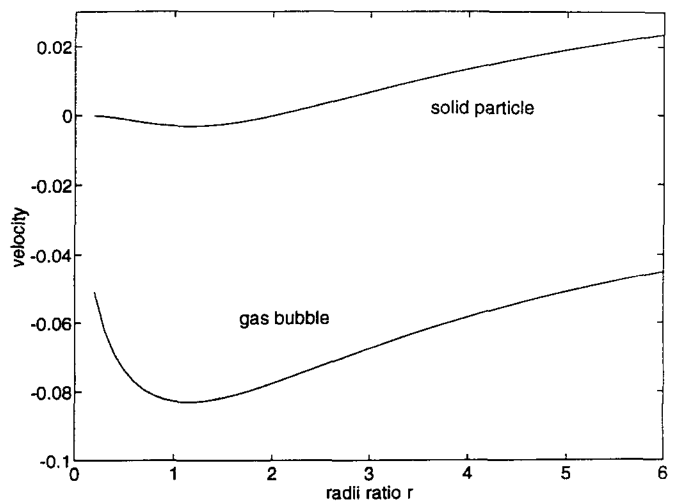

Figure 4. Dimensionless velocities of the solid particle and the gas bubble as functions of the bubble-to-particle radii ratio, $d = 0.1$.

linearly with its radius in accordance with the results of Young *et al.* (1959). With further growth of the radius, the bubble velocity reaches maximum and then diminishes tending to zero in the case of an infinitely large bubble. When r exceeds the threshold determined by d^*, the solid particle and the bubble begin to move towards each other. With the increase of bubble radius, the velocity of the solid particle increases and tends to a constant value corresponding to the interaction with a plane free surface. This case, however, requires special consideration and will be published elsewhere.

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